SNAP Centre Workshop

Graphing Lines

Graphing a Line Using Test Values

A simple way to linear equation involves finding test values, plotting the points on a coordinate plane, and connecting the points.

Example 1 Given the linear equation $y = 2x + 3$, find the four y-values that correspond to $x = 0, 1, 2, 3$.

Plot the resulting ordered pairs on the coordinate plane, then connect the plotted points to form a graph of solutions to the equation.

Start by substituting the x-values into the equation to find the corresponding y-values.

$y = 2x + 3$ $x = 0$

$y = 2(0) + 3$

$y = 3$ The resulting ordered pair is $(0,3)$

$y = 2x + 3$ $x = 1$

$y = 2(1) + 3$

$y = 2 + 3$

$y = 5$ The resulting ordered pair is $(1,5)$

$y = 2x + 3$ $x = 2$

$y = 2(2) + 3$

$y = 4 + 3$

$y = 7$ The resulting ordered pair is $(2,7)$

$y = 2x + 3$ $x = 3$

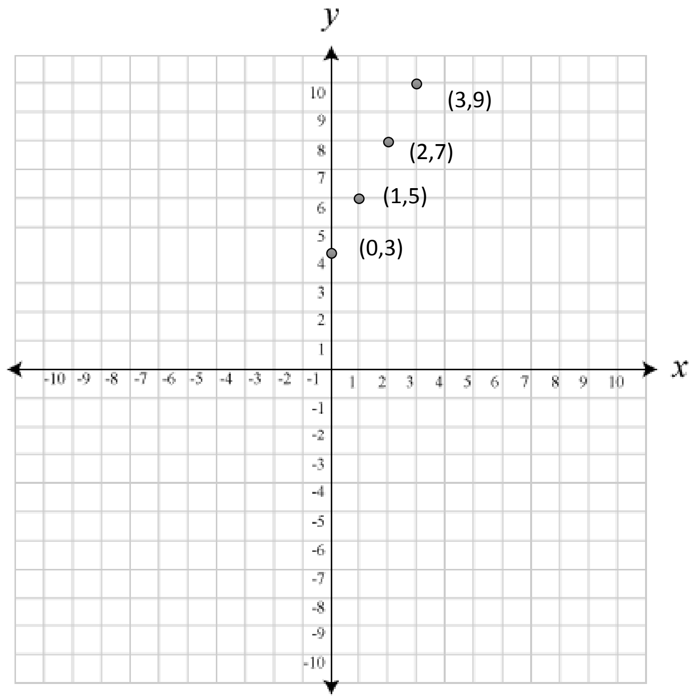
$y = 2(3) + 3$

$y = 6 + 3$

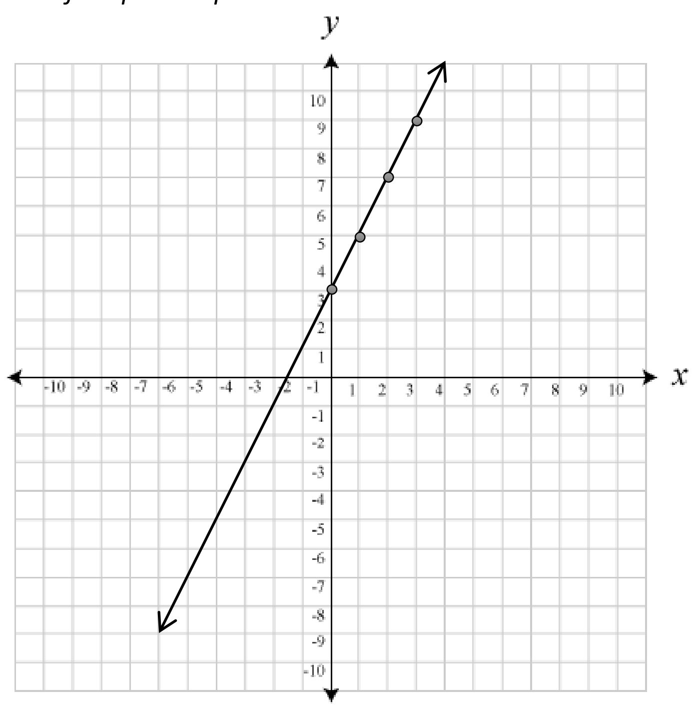
$y = 9$ The resulting ordered pair is $(3,9)$

Next, plot the ordered pairs on the coordinate plane.

*Conventionally, y-values are plotted on the vertical axis since y is the **dependent variable**, and x-values are plotted on the horizontal axis since x is the **independent variable**.*



Finally, draw a line through the four plotted points.



Example 2 Given the same equation, $y = 2x + 3$, find the two y-values that correspond to $x = -2, -1$. Again, plot the resulting ordered pairs on the coordinate plane, then connect the plotted points to form a graph of solutions to the equation.

Like the previous example, begin by substituting x-values into the equation to find corresponding y-values, and then construct ordered pairs.

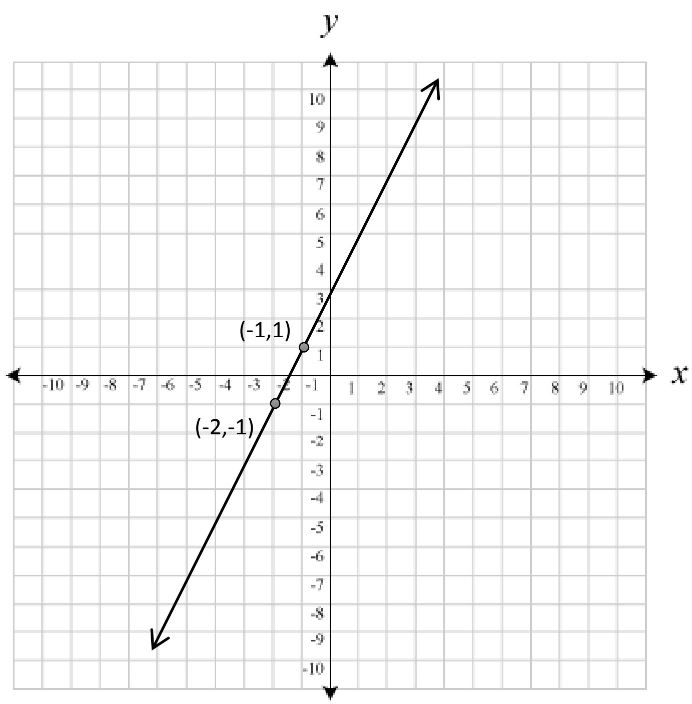
$y = 2(-2) + 3$ $\quad$ $x = -2$

$y = -1$ $\quad$ *The resulting ordered pair is* $(-2,-1)$

$y = 2(-1) + 3$ $\quad$ $x = -1$

$y = 1$ $\quad$ *The resulting ordered pair is* $(-1,1)$

Plot the new ordered pairs, taking note of the negative x- and y-values. Once the points are plotted, draw a line through them.



The line formed when plotting the second set of test values matches the line formed when plotting the first set. It doesn't matter which particular test values are used to graph a linear equation; as long as the points satisfy the conditions of the given equation, they can be found on the line.

Also, notice that only two points were needed to graph the equation. When using test values, **two points are needed to graph a line**. One point is not enough to define a line, and three or more points (as seen in **Example 1**) provide no more information regarding the line than two points would.

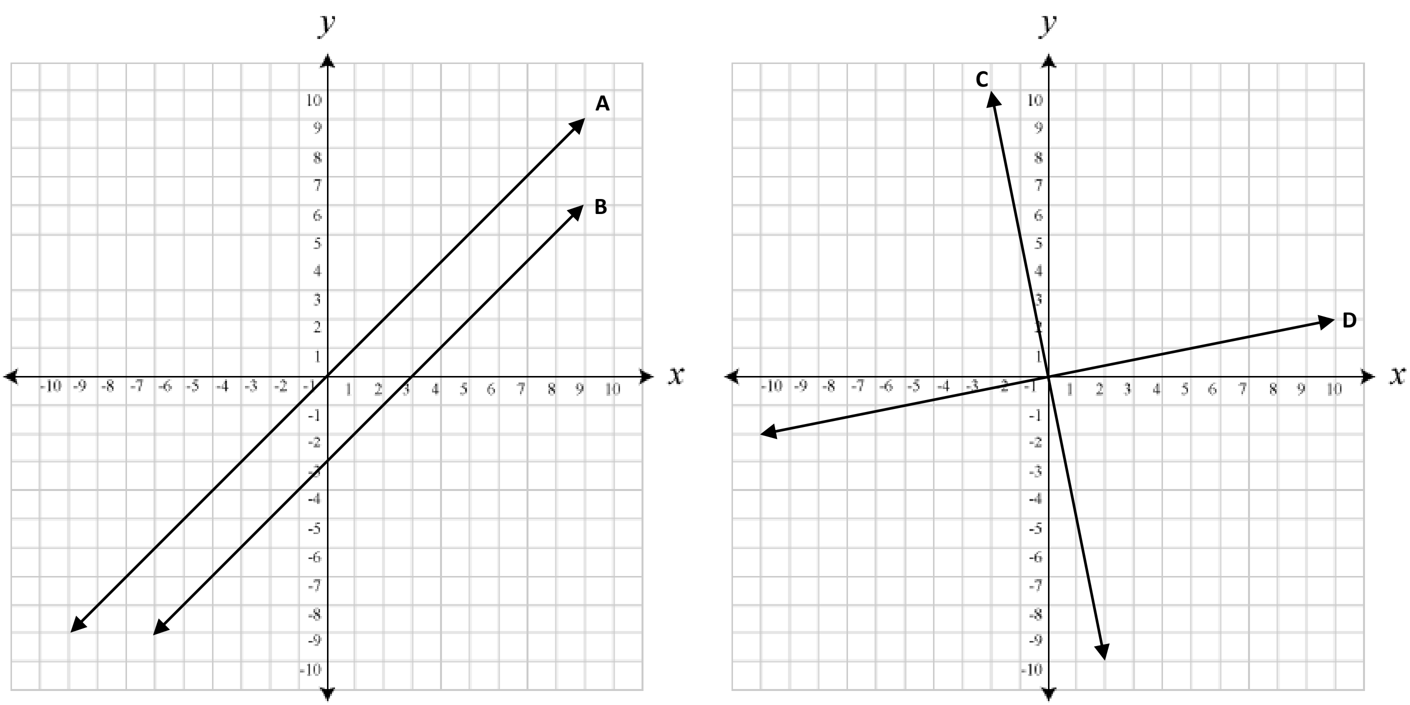
Slope of a Line

The slope of a line is an indicator of both steepness and direction, and can be used along with other pieces of information to graph a given line. It is defined as the ratio of a line's change in y-value to its change in x-value between any two points (x_1, y_1) and (x_2, y_2), and is found using the slope formula $\frac{y_2 - y_1}{x_2 - x_1}$.

Two common ways of expressing this ratio are $\frac{\Delta y}{\Delta x}$, with Δ ("delta") meaning "difference in" or "change in", and more casually $\frac{rise}{run}$ ("rise-over-run"), which attempts to physically describe the line's change along the vertical axis relative to its change along the horizontal axis.

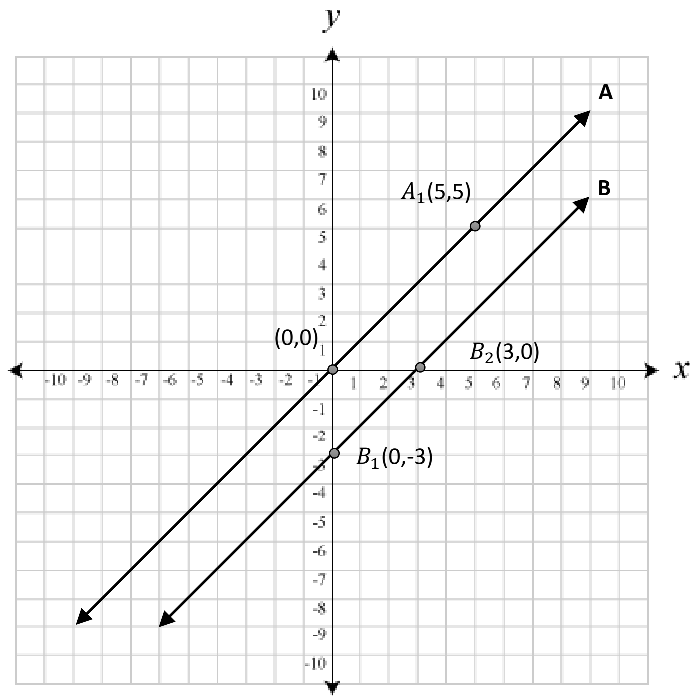
Traditionally, slope is represented by a lowercase m.

Example 3 Determine the slope of each of the lines pictured below.



Since the goal here is to measure the slope of each line, it is necessary to extract enough data to use the formula $\frac{y_2 - y_1}{x_2 - x_1}$*. That means for each line, two x- and y-values must be found.*

To find these values, plot any two points along each line. (If possible, try to pick points that correspond to whole numbers on the x- and y-axes.)



***Line A**: Line A passes through the origin, so a natural choice for one of the points is (0,0). Another point, A_1, is picked at random along the line at (5,5).*

Using the x- and y-values from these two points (y_1 = 5, y_2 = 0, x_1 = 5, x_2 = 0), the line's slope can be calculated:

$$m_A = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{0-5} = \frac{-5}{-5} = 1$$

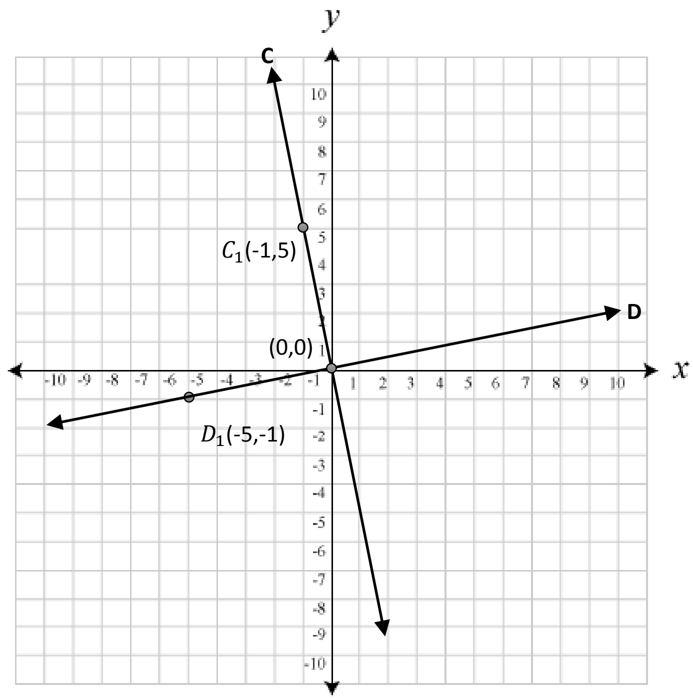
*Line A has a slope of +1. In terms of steepness, this indicates that for every unit the line moves along the x-axis, it moves that same number of units along the y-axis. **In terms of direction, a positive slope indicates that as the line increases along the x-axis, it also increases along the y-axis**.*

***Line B:** The points where line B crosses the x- and y-axes are suitable for calculating the slope, and are (3,0) and (0,-3), respectively.*

Again, using the corresponding x- and y-values (y_1 = -3, y_2 = 0, x_1 = 0, x_2 = 3), the line's slope can be calculated:

$$m_B = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-3)}{3-0} = \frac{3}{3} = 1$$

*Line B also has a slope of +1. Visually, this makes sense; both lines appear to be the same in terms of their steepness and direction. In fact, **any time two lines share the same slope and are not the same line, they are parallel, and will never intersect.***



Both lines pass through the origin, so only one additional point needs to be plotted for each line.

Line C*: The point (-1,5) is chosen which, along with (0,0) gives the values* y_1 *= 5,* y_2 *= 0,* x_1 *= -1,* x_2 *= 0*

$$m_C = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{0-(-1)} = \frac{-5}{1} = -5$$

Here, ***the negative slope indicates that as the line increases along the x-axis, it decreases along the y-axis.*** *Line C's slope has a magnitude of 5; for every unit line C increases along the x-axis, it decreases 5 units along the y-axis.*

Line D*: The points (-5,-1) and (0,0) on line D give the x- and y-values:* y_1 *= -1,* y_2 *= 0,* x_1 *= -5,* x_2 *= 0*

$$m_D = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-1)}{0-(-5)} = \frac{1}{5}$$

Line D has a positive slope that is less than 1. ***Keeping the slope in fraction form often makes it easier to see exactly how the line is behaving****; for every 5 units it increases along the x-axis, it increases 1 along the y-axis.*

Also, notice that lines C and D appear to be perpendicular, visually. In the same way lines that are parallel share the same slope, ***the slopes of perpendicular lines are the negative reciprocal of each other.*** *Accordingly, calculating the slope of line C as the negative reciprocal of line D gives the same result as calculating it using the slope formula:*

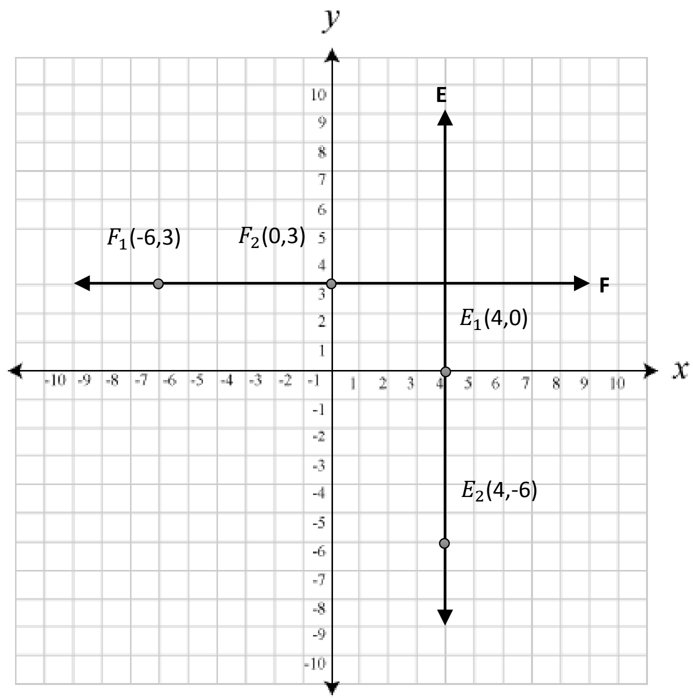
$$m_C = \frac{-1}{m_D} = \frac{-1}{(\frac{1}{5})} = -5$$

Slope of Horizontal and Vertical Lines

Examining line C illustrated that as the magnitude of a line's slope increases, its direction gets closer to becoming vertical. Similarly, examining line D illustrated that as the magnitude of a line decreases, approaching 0, its direction gets closer to becoming horizontal.

Next, the slope of lines that are completely horizontal or completely vertical will be examined.

Example 4 Determine the slope of each of the lines pictured below using the points given.



Line E: *The points* $(4,0)$ *and* $(4,-6)$ *on line E give the x- and y-values:* $y_1 = 0$, $y_2 = -6$, $x_1 = 4$, $x_2 = 4$

$$m_E = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(0)}{4-4} = \frac{-6}{0}$$

Calculating the slope for line E illustrates that (due to division by 0) ***the slope of a vertical line is undefined****.*

Line F: *The points* $(-6,3)$ *and* $(0,3)$ *on line F give the x- and y-values:* $y_1 = 3$, $y_2 = 3$, $x_1 = -6$, $x_2 = 0$

$$m_E = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{0-(-6)} = \frac{0}{6} = 0$$

Calculating the slope for line F illustrates that ***the slope of a horizontal line will be 0 divided by a non-zero number, which simplifies to 0****. No matter how far the line "runs" along the x-axis, there will be no "rise" along the y-axis.*

Point-Slope Form

Expressing the equation of a line using **point-slope form** involves rearranging the slope formula slightly, and substituting values into the equation for the line's slope and a single point, anywhere on the line.

Taking the line's slope, *m*, a point that is known on the line, (x_1, y_1), and the general point, (x, y), the slope formula can be rearranged as follows:

$$m = \frac{y - y_1}{x - x_1}$$

Multiplying both sides by $(x - x_1)$ *gives point-slope form.*

$$m(x - x_1) = y - y_1$$

It is more common, however, to see the equation reversed so that y-terms are on the left-hand side.

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

Remember, the values for m, x_1*, and* y_1 *are constant quantities when expressing the equation of a line in this form, while* x *and* y *remain as variables.*

Example 5 Express a line that has a slope m = -3, and passes through the point (-1, 7) in point-slope form.

Our x_1 *and* y_1 *values are given by the by the point* $(-1,7)$*, and our slope is given explicitly. All we need to express the equation of this line in point-slope form is substitute this information into the point-slope equation and simplify.*

$$y - 7 = -3(x - (-1))$$

$$\boldsymbol{y - 7 = -3(x + 1)}$$

Slope-Intercept Form

Expressing the equation of a line using **slope-intercept form** also involves rearranging the slope formula slightly, and values for the line's slope and single point on the line are substituted into the equation, however, the point cannot be chosen anywhere along the line; the point used is where the line crosses the y-axis, also known as the **y-intercept**.

Using the fact that the x-value of a point where a line crosses the y-axis must be equal to 0, the coordinates of the y-intercept are taken to be (0,b), where b represents the constant y-value of the y-intercept for a given line.

Using this, as well as the line's slope, *m*, and the general point (*x,y*), the slope formula can be rearranged as follows:

$$m = \frac{y - b}{x - 0} = \frac{y - b}{x}$$

After disregarding the 0 in the denominator, multiply both sides by x **.**

$$mx = y - b$$

Add b to both sides to isolate y.

$$mx + b = y$$

This is the final equation, however, slope-intercept form is far more commonly arranged with the y on the left-hand side.

$$\boldsymbol{y = mx + b}$$

When using slope-intercept form, m and b are constants, whereas x and y still remain as variables. Also, it is important to make sure y is fully isolated.

Example 7 A line has a slope m = 1/3, and crosses the y-axis at (0,9). Express the equation for this line in slope-intercept form.

All of the required information is given, all we need to do is substitute it into the slope-intercept equation.

$$\boldsymbol{y = \frac{x}{3} + 9}$$

Example 8 A line crosses the y-axis at the point (0,-5), and passes through (2,1). Express the equation for this line in slope-intercept form and point-slope form

A good place to start in this example is calculating the slope, since it will be used for both forms.

$$m = \frac{-5-1}{0-2} = \frac{-6}{-2} = 3$$

Now that we have our slope, the equation can be expressed in either form since we have our y-intercept.

$$y = 3x - 5 \qquad Slope - intercept\ form.$$

$$y - 1 = 3(x - 2) \qquad Point - slope\ form.$$